

M.I. Maximum Revenue

Do Now:

Given the system of 2 equations, use algebra to create one equation in standard form.

$$3x^2 + 2y^2 - 54y - 143 = 0$$

$$x = 3y + 3$$

$$3(3y+3)^2 + 2y^2 - 54y - 143 = 0$$

$$3(9y^2 + 18y + 9) + 2y^2 - 54y - 143 = 0$$

$$\rightarrow 27y^2 + 54y + 27 + 2y^2 - 54y - 143 = 0$$

$$29y^2 - 116 = 0$$

$$y^2 - 4 = 0$$

$$(y+2)(y-2) = 0$$

$$y = -2 \quad | \quad y = 2$$

$$\swarrow$$

$$x = ?$$

$$\searrow$$

$$x = ?$$

- What is **revenue**?

the amount of \$ you bring in
as a result of sales.

The revenue of a product or service can depend on its price in two ways.

1. An increase in price means that more revenue per unit is earned but less units are sold.
2. A decrease in price means that less revenue is earned per unit but more units are sold.

In the next problems, a current price and sales amount will be given and how a price increase or decrease will affect the sales level.

- A cinema multiplex averages 2000 tickets sold on a Saturday when ticket prices are \$8. A research firm has determined that for each \$0.25 increase in the ticket price, 50 fewer tickets will be sold. revenue \$16,000

What is the maximum revenue and what ticket price maximizes the revenue?

$\$16,087$ \times 1 increase = $\$8.25$, $\frac{1950 \text{ people}}$
 $\$16,150...$ \times 2 increases = $\$8.50$, $\frac{1900}{}$
 ...

Let x = # of \$0.25 ticket increases

$8 + .25x$ = Ticket price ← rate of change (slope)
 $2000 - 50x$ = Avg # of Tickets sold ← rate of change

To Get Revenue: $\left(\begin{matrix} \text{multiply} \\ \# \text{ of} \\ \text{tickets} \end{matrix} \right) \times \left(\begin{matrix} \text{cost/price} \\ \text{per} \\ \text{ticket} \end{matrix} \right)$

- Ticket Revenue:

$$R(x) = (2000 - 50x)(8 + .25x)$$

$$= 16000 + 500x - 400x - 12.5x^2$$

$$R(x) = -12.5x^2 + 100x + 16,000$$

↑ Used to determine the max revenue as a function of the # increases (x).

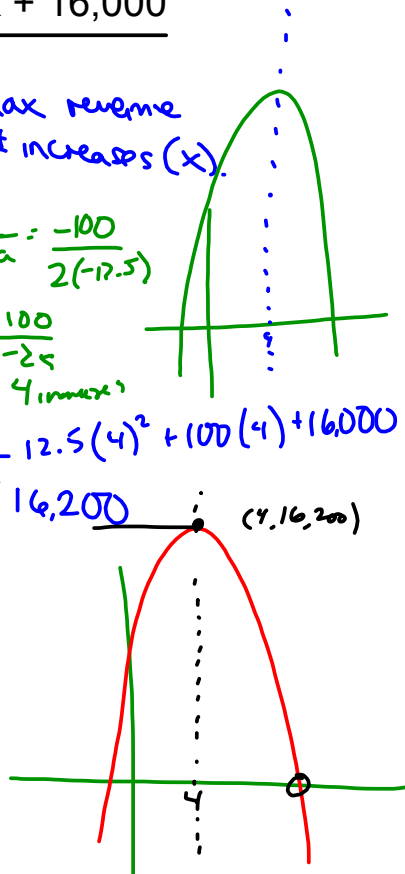
A.O.S. = $x = -\frac{b}{2a} = \frac{-100}{2(-12.5)}$

$x = \frac{-100}{-25}$
 $x = 4, \text{max}$

$$R(4) = -12.5(4)^2 + 100(4) + 16,000$$

$$= \$16,200$$

If we raise the ticket price 4 times to \$9.00, then we will get a max revenue of \$16,200.



The John Deere company has found that the revenue from sales of heavy-duty tractors is a function of the unit price p that it charges. The revenue R is

$$R = -\frac{1}{2}p^2 + 1900p$$

What unit price p should be charged to maximize revenue? What is the maximum revenue?

A management firm has determined that 60 apartments in a complex can be rented if the monthly rent is \$900, and that for each \$50 increase in the rent, three tenants are lost with little chance of being replaced.

What rent should be charged to maximize revenue?

What is the maximum revenue?

let $x = \#$ of \$50 increases

• let $\frac{60 - 3x}{}$ = avg # tenants

• let $\frac{900 + 50x}{}$ = avg cost per apartment

$$R(x) = (60 - 3x)(900 + 50x)$$

The price p and the quantity x sold of a certain product obey the demand equation below.

$$x = -7p + 112, \quad 0 \leq p \leq 20$$

- (a) Express the revenue R as a function of x .
- (b) What is the revenue if 63 units are sold?
- (c) What quantity x maximizes revenue? What is the maximum revenue?
- (d) What price should the company charge to maximize revenue?
- (e) What price should the company charge to earn at least \$336 in revenue?